Coherence in semiconductor nanostructures Part IV: Notions of nonlinear spectroscopy & a particular experimental realization

Jacek Kasprzak



Equipe mixte CEA-CNRS "Nanophysique et semicondcuteurs" Institut Néel - CNRS Grenoble France

Warsaw University, October-December 2020

Exploring coherence in solids







2 Nonlinear optical responses

- Second- Third- & High-Harmonic Generation
- **3** Relevance of FWM



$\begin{array}{l} \mbox{Linear Response} \\ \mbox{Polarization} \propto \mbox{EM field} \end{array}$

• A solution of Maxwell's equations:

$$\varepsilon(\mathbf{r},t) = \varepsilon e^{i(\mathbf{kr}-\omega t)} + c.c. \tag{1}$$

• Induced macroscopic polarization:

$$P(\mathbf{r}, t) = \chi(\mathbf{r}, t)\varepsilon(\mathbf{r}, t)$$
(2)

- $\chi(\mathbf{r}, t)$: first-order susceptibility \Rightarrow response of the matter to an electromagnetic field.
- The dielectric function ϵ and the complex refractive index *n* are related to it by:

$$\epsilon(\mathbf{r},t) = \epsilon_0 (1 + \chi(\mathbf{r},t)), \ n = \sqrt{1 + \chi(\mathbf{r},t)}$$
(3)

Absorption, Reflectance

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Optical Nonlinear Phenomena Induced polarization P scales with the 2nd, 3rd, ...M-th power of the impinging field \mathcal{E}

$\chi^{(2)}$ process	$\chi^{(3)}$ process	Higher order process
sum frequency generation	four-wave mixing	<i>n</i> -wave mixing
difference frequency generation	coherent Raman scattering	mode locking
optical retrification	phase modulation/conjugation	generation of continuum
parametric scattering	optical solitons	n-photon absorption



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VOLUME 7, NUMBER 4

PHYSICAL REVIEW LETTERS

AUGUST 15, 1961

GENERATION OF OPTICAL HARMONICS*

P. A. Franken, A. E. Hill, C. W. Peters, and G. Weinreich The Harrison M. Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan (Received July 21, 1961)





34 35 36 37 38 39 40 **45 50 55 60 65 70 75 80**

FIG. 1. A direct reproduction of the first plate in which there was an indication of second harmonic. The wavelength scale is in units of 100 A. The arrow at 3472 A indicates the small but dense image produced by the second harmonic. The image of the primary beam at 6943 A is very large due to halation.

Basics





2 Nonlinear optical responses

• Second- Third- & High-Harmonic Generation





Harmonic Generation in MoS_2

Multi-photon microscope: exciting $\hbar\omega$, detecting $2\hbar\omega$, $3\hbar\omega$, $5\hbar\omega$







Fig. 3 Multiphoton images of MoS_2 flakes. a SHG and b THG map of the flakes in Fig. 1a. c Optical spectrum of the nonlinear signal from 1L-MoS₂ with a peak irradiance ~30 GW cm⁻²

Fig. 2 Schematic diagram of multiphoton microscope. MLL, linearly

Harmonic Generation in MoS_2

Multi-photon microscope: exciting $\hbar\omega$, detecting $2\hbar\omega$, $3\hbar\omega$, $5\hbar\omega$

A. Säynätjoki et al.Nature Communications 8, 893 (2017)



High-resolution imaging via multi-photon microscopy

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High-resolution imaging via multi-photon microscopy

High-Harmonic Generation in MoS₂

Femto-second excitation: exciting $\hbar\omega=0.3\,{
m eV}$, detecting up to $13\hbar\omega$





Frequency-tuned Second Harmonic Generation Level structure of excited states in WSe₂ monolayer





Basics

Toward the four-wave mixing

• In case of pair of fields:
$$\varepsilon(\mathbf{r},t) = \varepsilon_1(\mathbf{r}_1,t) + \varepsilon_2(\mathbf{r}_2,t- au)$$
, one has:

$$P^{(\mathrm{N})} \propto \varepsilon_1^{a_1} \varepsilon_1^{* \, b_1} \varepsilon_2^{a_2} \varepsilon_2^{* \, b_2} \tag{4}$$

• For N = (0, 1, 2, 0), the resulting 3rd order polarization is:

$$\mathsf{P}^{(3)} \propto R^{(3)}(\omega_3, t) \varepsilon_1^* \varepsilon_2^2 e^{-i(\mathbf{k}_1 \mathbf{r} + \omega_1 t)} e^{2i(\mathbf{k}_2 \mathbf{r} + \omega_2 t)} \propto \\ \varepsilon_1^* \varepsilon_2^2 e^{+i[(2\mathbf{k}_2 - \mathbf{k}_1)\mathbf{r} + (2\omega_2 - \omega_1)t]}, (5)$$

• Such $P^{(3)}$ propagates onto the $2\mathbf{k}_2 - \mathbf{k}_1$ direction and oscillates at $2\omega_2 - \omega_1$ frequency, and is called degenerate **four-wave mixing** or FWM.





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Why it is worth to look into FWM $\propto \mathcal{E}_1^* \mathcal{E}_2 \mathcal{E}_2$? Because it offers an access to homogeneous dephasing time $T_2 = 2\hbar/\gamma$ in the presence of spectral inhomogeneous broadening σ via formation of a photon echo



Rephasing of all polarizations at $t = 2\tau \Rightarrow$

FWM is only sensitive to microscopic dephasing, independent of σ . σ is inferred through the time-spread of the echo.

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FWM retrieval by spectral interference of the reference field with the heterodyne beat at the FWM frequency

 $E_{FWM} \propto \exp[i(\mathbf{k_{FWM}x} - \omega_{FWM}t)]$

distinct by directions - ${f k}$ \Rightarrow ω - distinct in frequency spatial homogeneity - $\mathbf{X} \Longrightarrow \mathbf{t}$ - temporal invariance





FWM retrieval by spectral interference of the reference field with the heterodyne beat at the FWM frequency

Optical Heterodyning



Lets explain this sentence

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? How to extract FWM of a localized state ? Microscopy required \Rightarrow colinear arrangement of $\mathcal{E}_{1,2,3} \Rightarrow$ Optical Heterodyning



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Heterodyne FWM in 3 equations

• Pulse train: N delta pulses centered at ω with the repetition rate τ^{-1} :

$$\mathcal{E}(t) = A(t)e^{-i\omega t} + A(t-\tau)e^{-i\omega t} + A(t-2\tau)e^{-i\omega t} + \dots = \sum_{n=0}^{N} A(t-n\tau)e^{-i\omega t}$$

• One needs a proper phase shifter at frequency θ acting on a pulse train:

$$\mathcal{E}(t) = A(t)e^{-i\omega t} + A(t-\tau)e^{-i(\omega t+\theta \tau)} + ... = e^{-i\omega t}\sum_{n=0}^{N} e^{-in\theta \tau}A(t-n\tau)$$

• Induced FWM response:

$$\begin{split} R^{(-1,2)}(t) &\propto \mathcal{E}_{1}^{*}(t)\mathcal{E}_{2}^{2}(t) = e^{i\omega t} \sum_{n} A_{1}^{*}(t-n\tau) e^{in\theta_{1}\tau} e^{-2i\omega t} \sum_{m} A_{2}^{2}(t-m\tau) e^{-2im\theta_{2}\tau} = \\ e^{-i\omega t} \sum_{n} A_{1}^{*}(t-n\tau) A_{2}^{2}(t-n\tau) e^{-in(2\theta_{2}-\theta_{1})\tau} = e^{-i\omega t} \sum_{n} A_{FWM}(t-n\tau) e^{-in\theta} FWM^{\tau} \end{split}$$

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FWM micro-spectroscopy \Rightarrow optical lock-in





3-beam heterodyne detection & spectral interferometry Measurement of the exciton polarization and density dynamics with an enhanced spatio-temporal resolution: (100 fs, 300 nm)

Exploring coherence in solids

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W. Langbein et al.Optics Letters 31, 1151 (2006), intensly exploited in

Grenoble



3-beam heterodyne detection & spectral interferometry Measurement of the exciton polarization and density dynamics with an enhanced spatio-temporal resolution: (100 fs, 300 nm)

Exploring coherence in solids

FWM micro-spectroscopy \Rightarrow in practice Impact of an Acousto-Optic Modulator on relative phases of the consecutive pulses within a pulse train













































P_{a,b} contains interference term

 $2P_{a,b}(\omega,t) = |E_r|^2 + |E_s|^2 \pm 2\Re(E_r \cdot E_s^* \cdot e^{i\Omega_D t})$



Balanced detection is filtering the signal at Ω_D $P_D(\omega) = P_a - P_b = 2 \int_0^T \Re(E_r \cdot E_s^* \cdot e^{i\Omega_D t}) dt$



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Detection Scheme Balanced detection in a mixing AOM pump-reference interference





$\begin{array}{c} \mbox{Detection Scheme} \\ \mbox{Intereference period vs. delay,} \propto \tau^{-1} \end{array}$





$$P_D(\omega) = 2 \int_0^T \Re(E_r \cdot E_s^* \cdot e^{i\Omega_D t}) dt$$
 jest rzeczywista



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Transformata Fouriera do t

 $F^{-1}(P_D)$ zawiera 2 czasowo odwrócone składniki



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Transformata Fouriera do t

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Zasada przyczynowości

Tylko dodatnie czasy są fizyczne: $\Theta(t)[F^{-1}(P_D)]$





Photos of the setup Acousto-Optic Modulators used as frequency shifters



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